

Impulsions lumineuses ultracourtes et applications en diagnostic optique

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- Ultrashort pulses – some orders of magnitude
- Ultrashort pulse generation : Mode-locked oscillators
- Frequency comb spectroscopy
- Energy scaling
- Coherent Raman spectroscopy in reactive media
- Ultrafast imaging

Impulsions gaussienne : E(t) champ électrique

Domaine temporel :

$$E(t) = E_0 \exp\left(-\frac{t^2}{2T_0^2}\right) \exp(-i\omega_0 t)$$



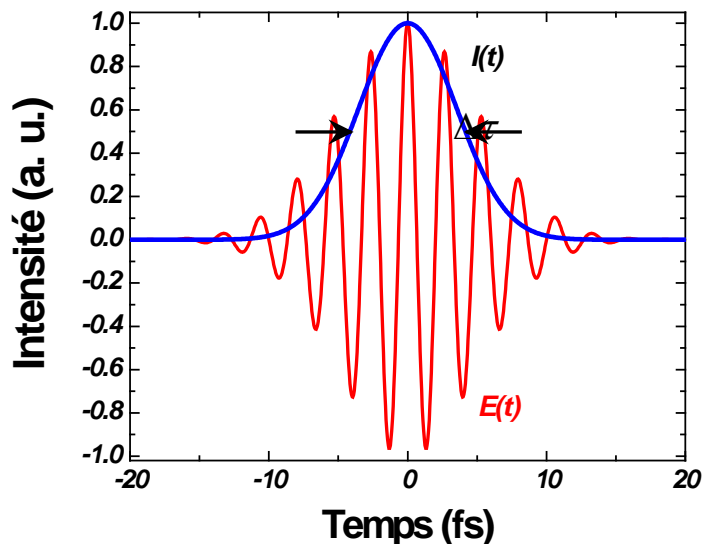
Domaine fréquentiel :

$$E(\omega) = E_0 T_0 \sqrt{2\pi} \exp\left(-\frac{(\omega - \omega_0)^2 T_0^2}{2}\right)$$

$$I(t) = I_0 \exp\left(-\frac{t^2}{T_0^2}\right)$$

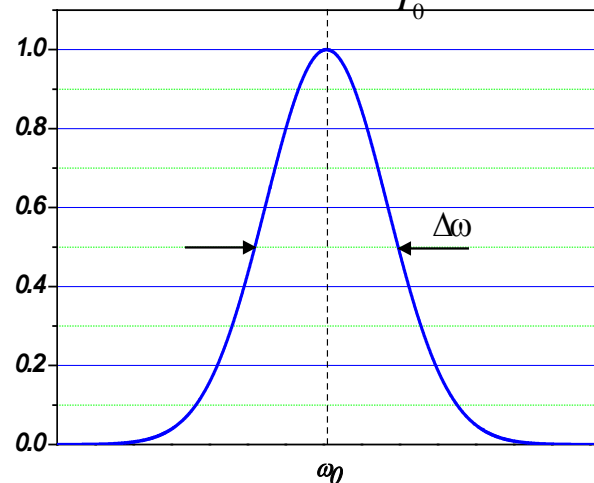
$$I(\omega) = 2\pi T_0^2 I_0 \exp\left(-(\omega - \omega_0)^2 T_0^2\right)$$

Largeurs à mi-hauteur : $\Delta\tau = 2\sqrt{\ln 2} T_0$

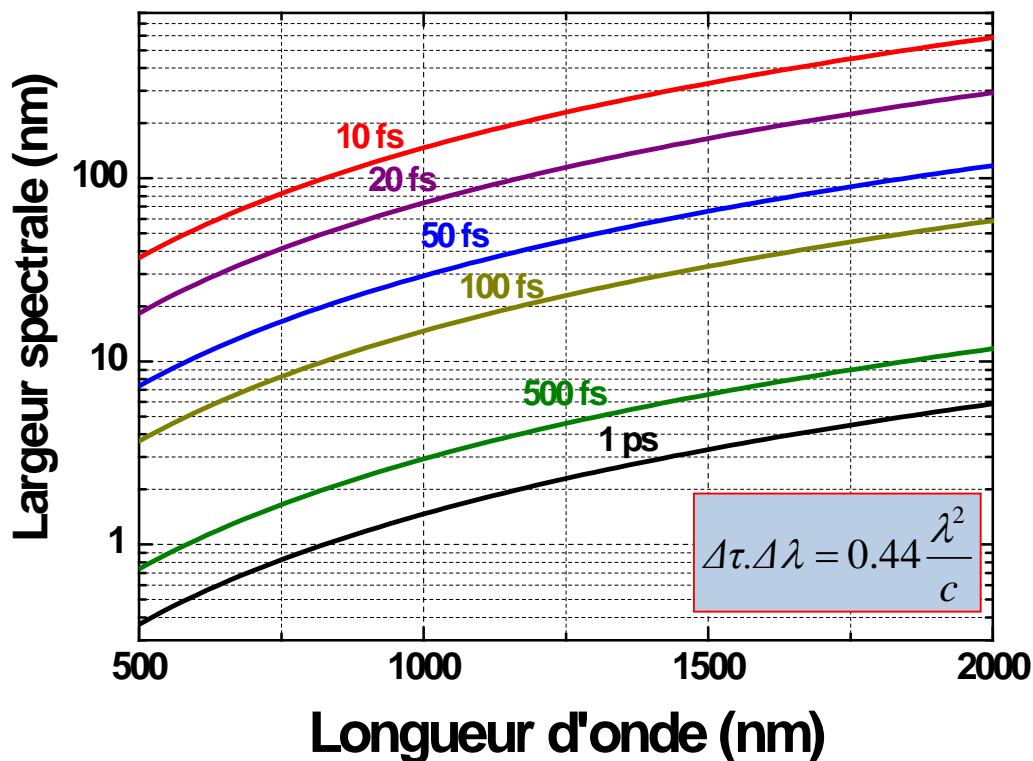


$$\Delta\tau \Delta\nu = \frac{2 \ln 2}{\pi} = 0.44$$

$$\Delta\omega = \frac{2\sqrt{\ln 2}}{T_0}$$



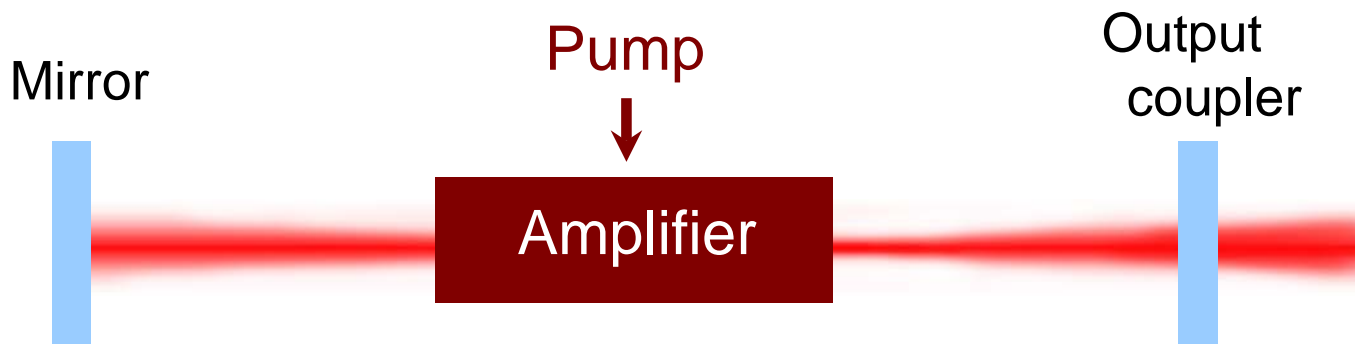
Ultrashort pulses \Rightarrow Large bandwidth



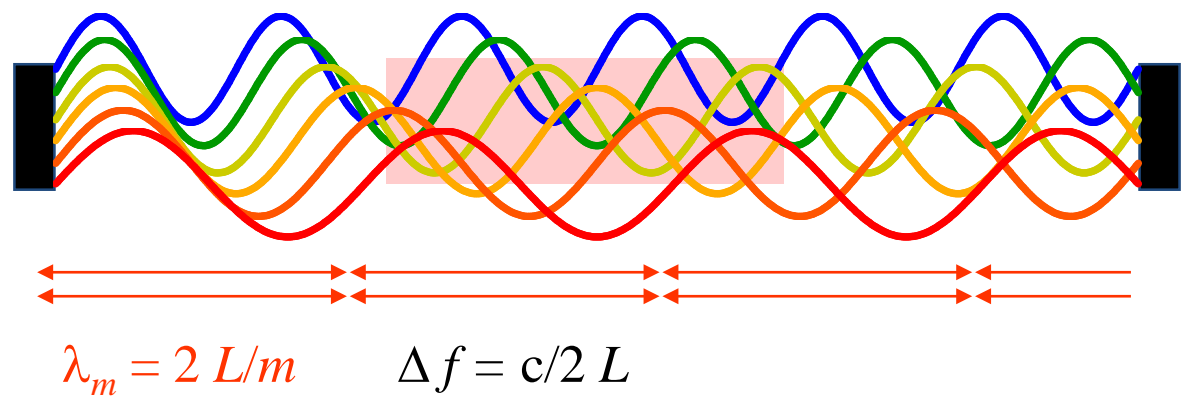
$\Delta\tau \cdot \Delta\nu = K$
 $K = 0.44$: Gaussian pulse
 $K = 0.315$: sech² pulse

Gaussian pulse width 100 fs at 1 μm \Rightarrow spectral width 14.6 nm

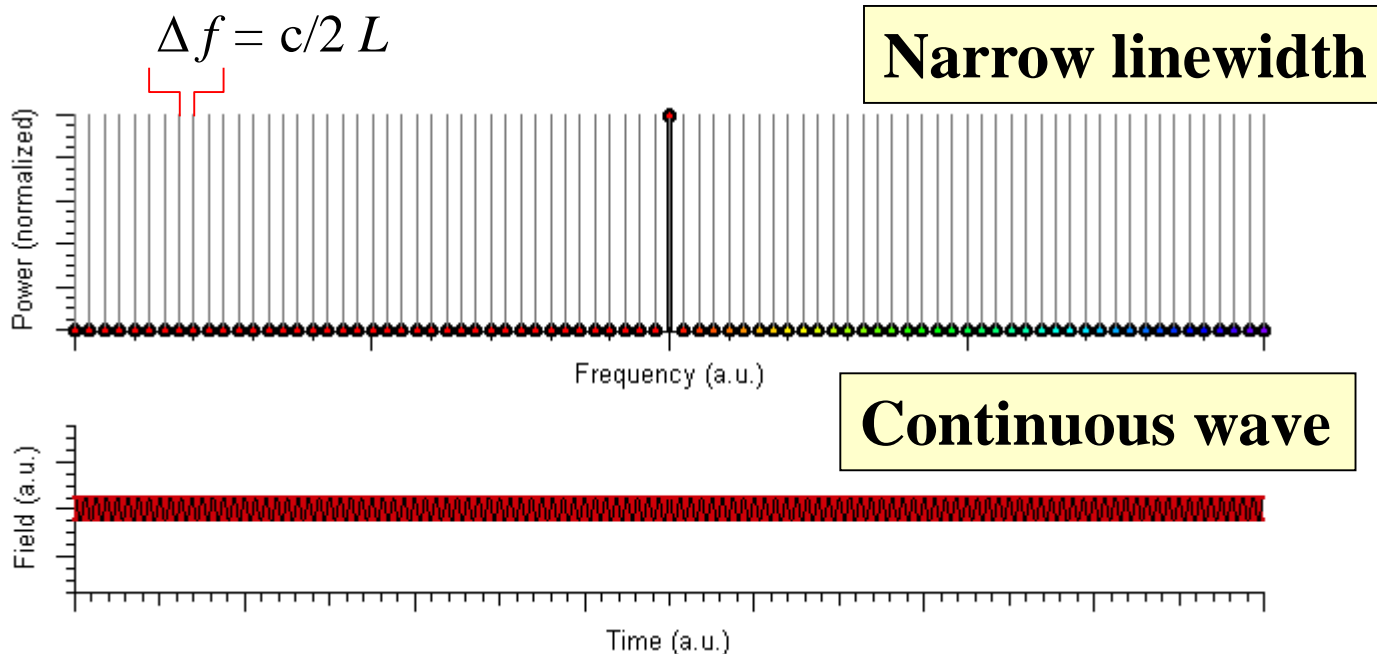
Pulse width 20 fs \Rightarrow spectral width 73 nm



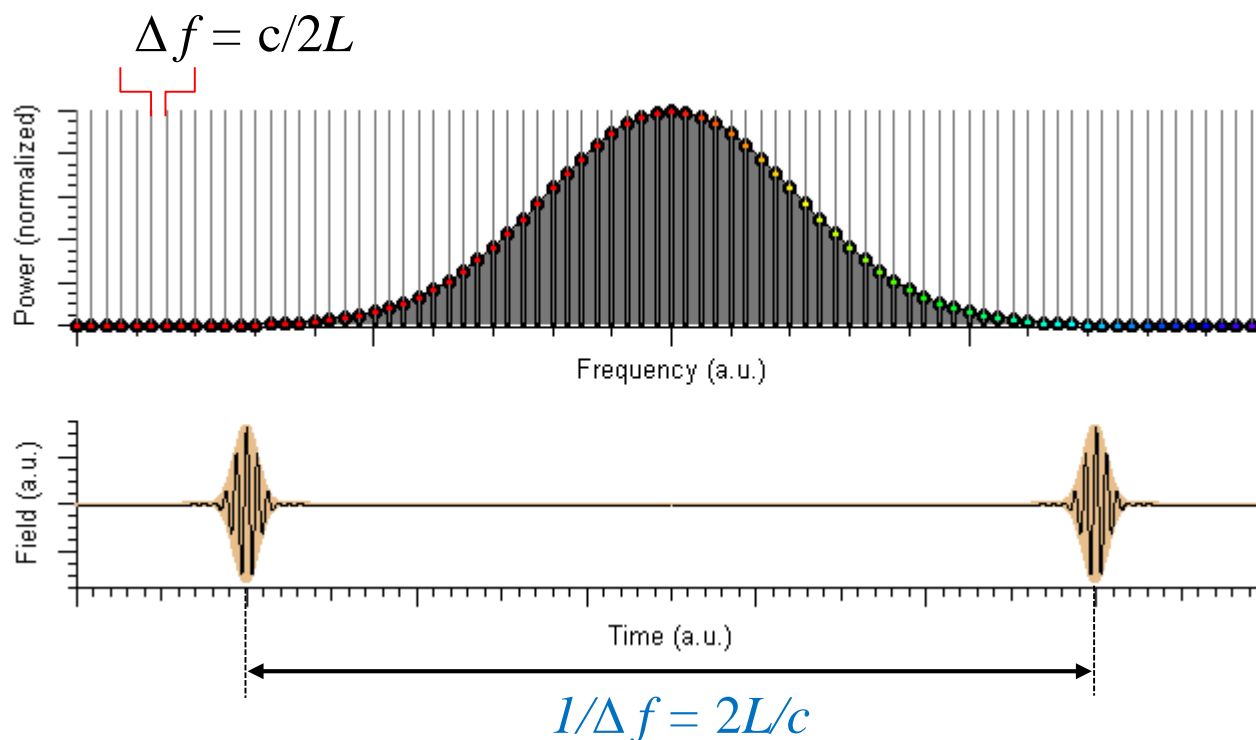
Longitudinal modes – spatial presentation



Time-bandwidth relation

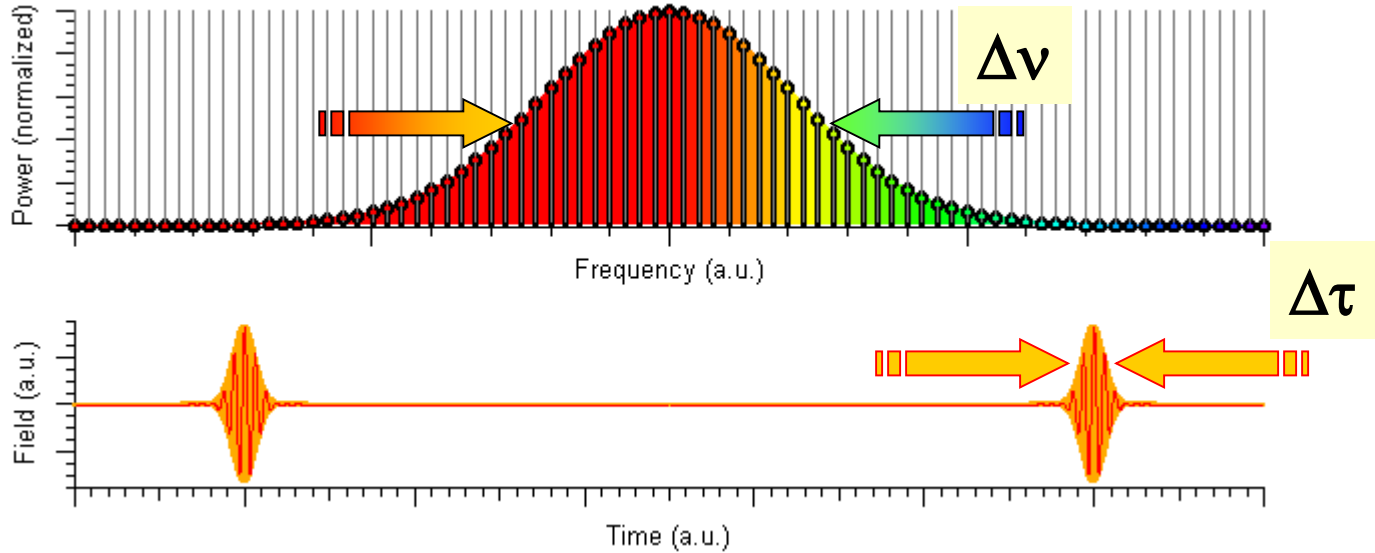


Time-bandwidth relation



Need to phase a large number of spectral components.

Time-bandwidth relation



$$\Delta\tau \cdot \Delta\nu = K$$

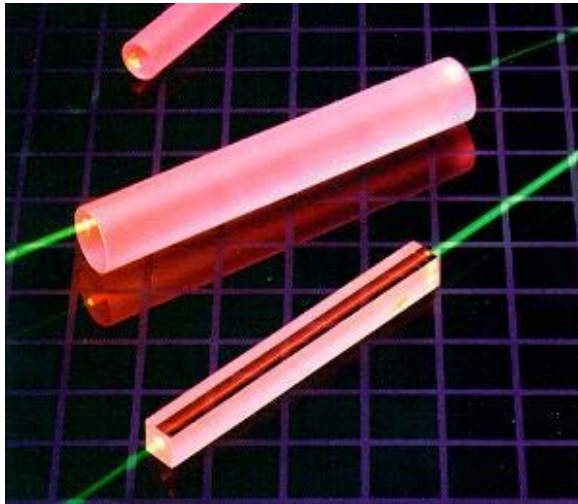
$K = 0.44$: Gaussian pulse

$K = 0.315$: sech^2 pulse

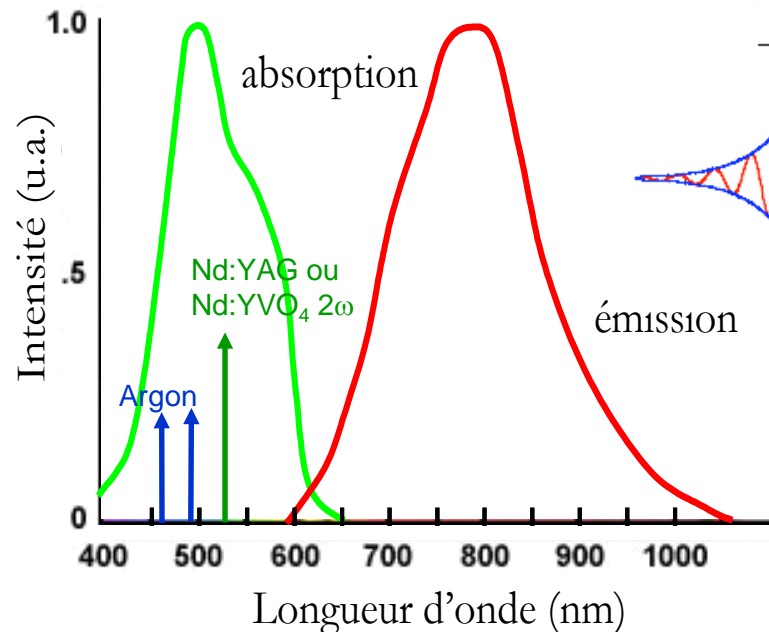
Need for large bandwidth amplifiers

Best performances with Titanium-doped sapphire crystals $\text{Ti}^{3+}:\text{Al}_2\text{O}_3$

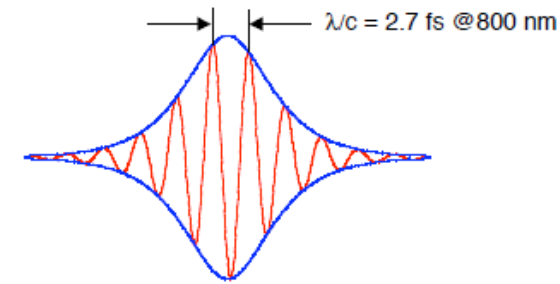
175 nm corresponds to **5,3 fs** at 800 nm
 ≈ Duration ready available from Ti:Sa oscillators based



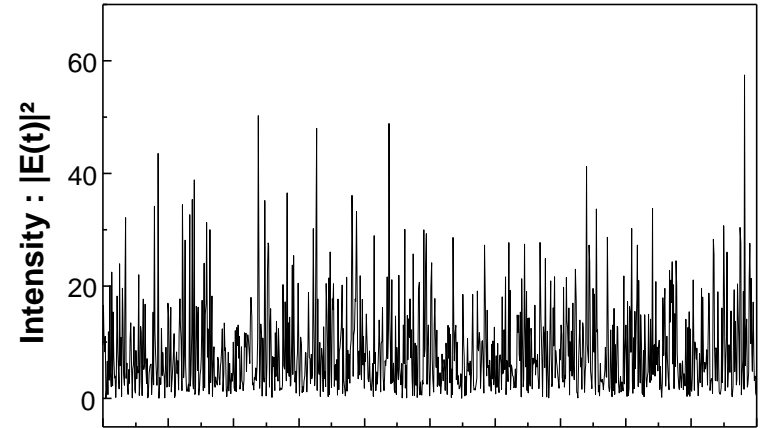
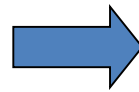
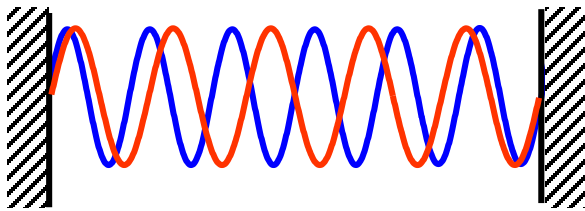
Ti^{3+} doping concentration
 : 1 % weight



NB : 1 optical cycle



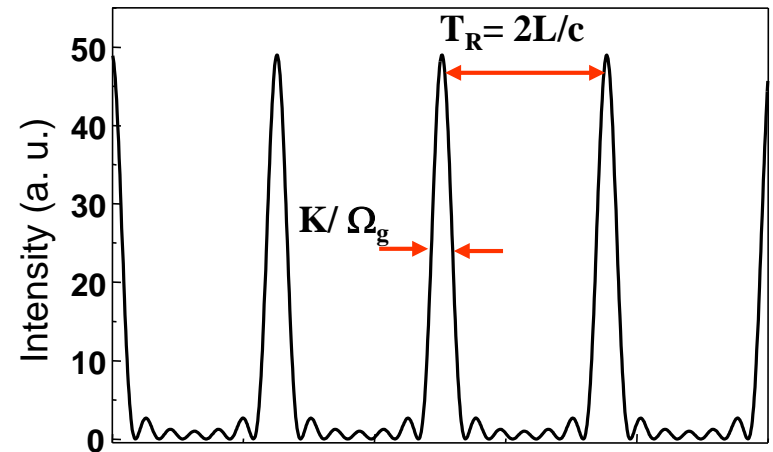
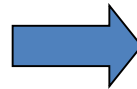
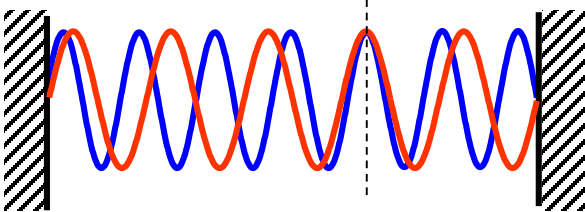
Random phase modes :

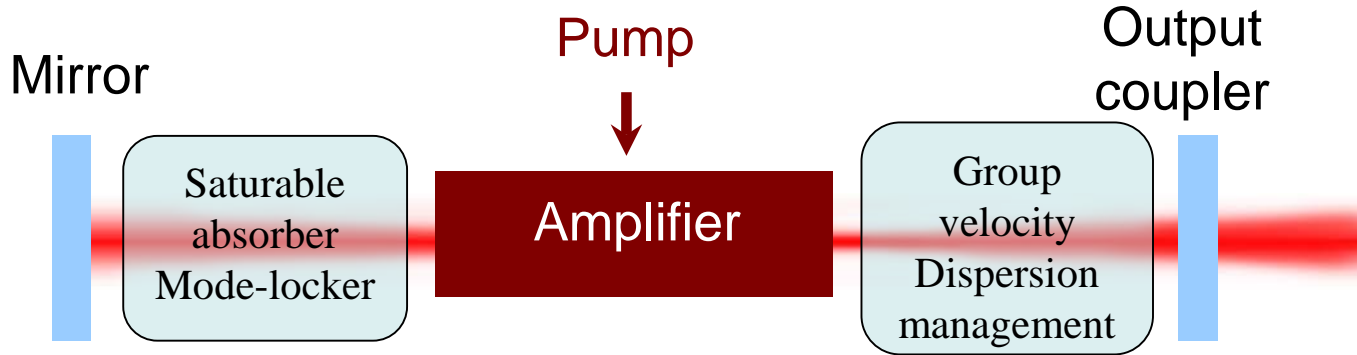


In phase modes

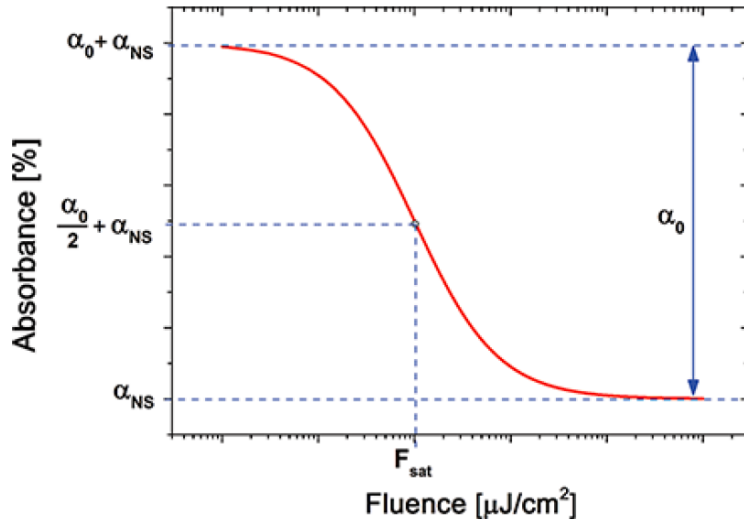
$$\Phi_m = 0, \forall m \Rightarrow$$

$$I(t) = I_0 \frac{\sin(N\omega_{isl}t/2)}{\sin(\omega_{isl}t/2)}$$





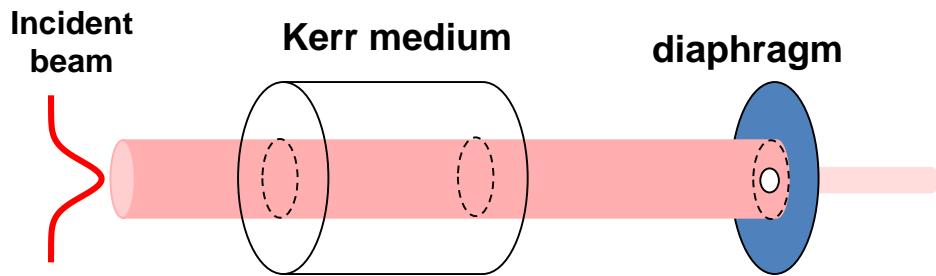
- Saturable absorber : promote pulsed operation against cw.
- High-intensity spikes burn through; low-intensity light is absorbed.



Several saturable absorbers:

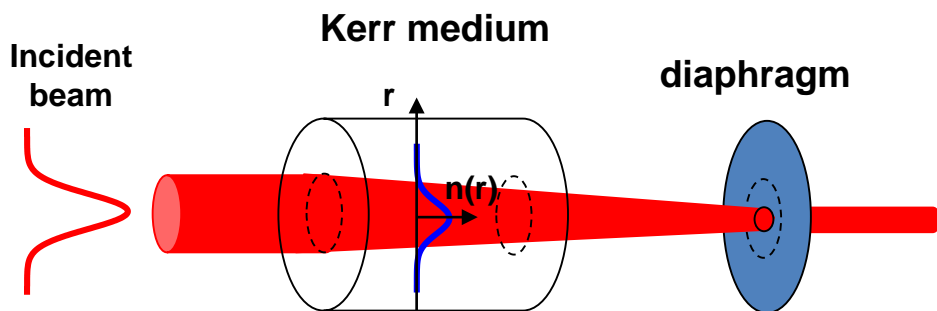
- Dye saturable absorbers
- Semiconductors (SESAMs)
- Graphene and carbon nanotubes
- Kerr-based saturable absorbers
- ...

Kerr-lens mode-locking (KLM)

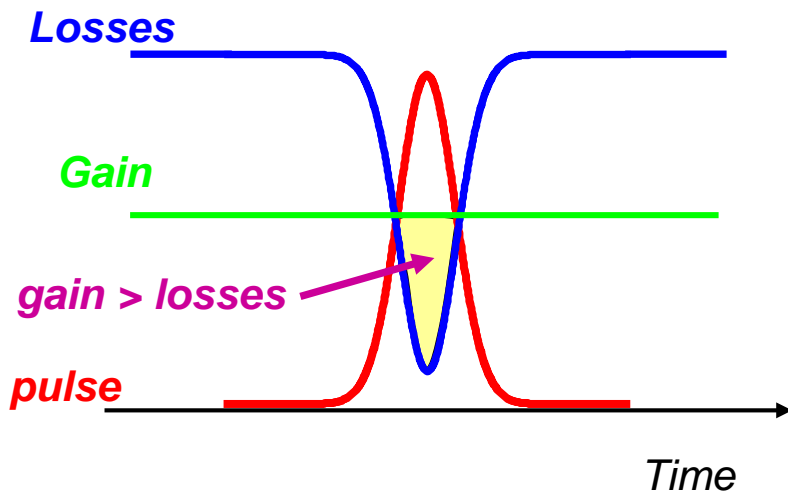


➤ refractive index of the medium varies with intensity : $n(I) = n_0 + n_2 I$

Low intensity : low transmission



Fast saturable absorber

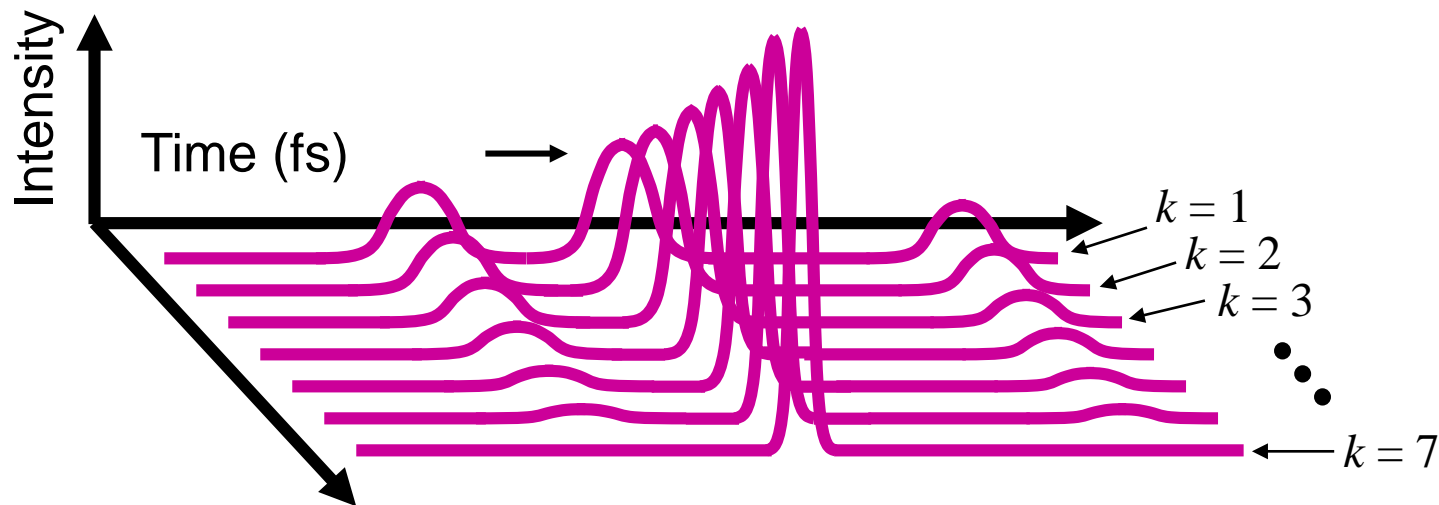


High intensity : high transmission

D. E. Spence, P. N. Kean, W. Sibbett, Opt. Lett. 16, 42, 1991

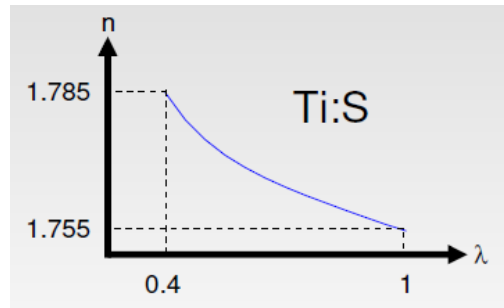
The pulse construct from noise (ns-ps):

- The SA imposes high losses for low intensity structures
- The high-intensity noise structure is shortened after several round-trips
- Intermodals coherence constructs naturally!



Propagation of broadband pulses \Leftrightarrow group velocity dispersion

- Refractive index varies with frequency



$$k(\omega) = n(\omega) \frac{\omega}{c}$$

$$k(\omega) = k_0 + \underbrace{\left. \frac{\partial k}{\partial \omega} \right|_{\omega_0}}_{\text{Group velocity}} (\omega - \omega_0) + \frac{1}{2} \underbrace{\left. \frac{\partial^2 k}{\partial \omega^2} \right|_{\omega_0}}_{\text{2nd order GVD}} (\omega - \omega_0)^2 + \frac{1}{6} \underbrace{\left. \frac{\partial^3 k}{\partial \omega^3} \right|_{\omega_0}}_{\text{3rd order dispersion}} (\omega - \omega_0)^3 + \dots$$

$$\equiv 1 / v_g$$

Group velocity

$$\equiv \beta_2 [ps^2 / m]$$

2nd order GVD

$$\equiv \beta_3 [ps^3 / m]$$

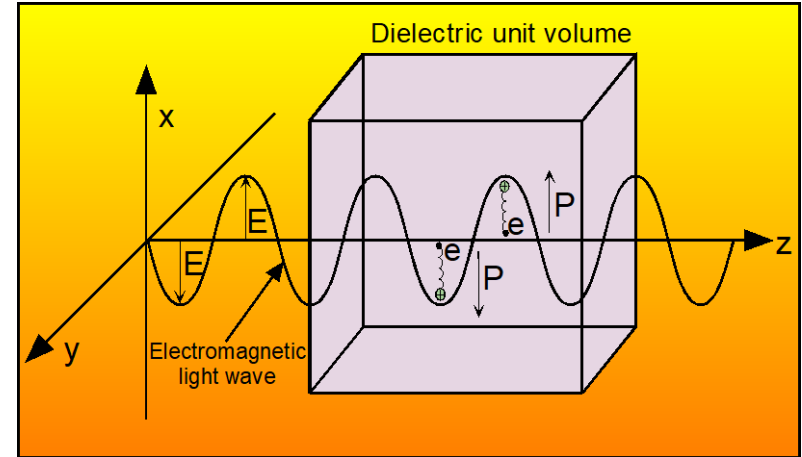
3rd order dispersion

Accumulated phase:

$$\varphi(\omega) = k(\omega) \cdot d = \varphi_0 + \varphi^{(1)} (\omega - \omega_0) + \varphi^{(2)} (\omega - \omega_0)^2 + \dots$$

Propagation of intense pulses : nonlinear polarization component induced in the medium

$$P = \epsilon_0(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \dots)$$



$\chi^{(1)}$
Linear susceptibility
 ↓
 Classical optical effects
 (reflection, absorption)

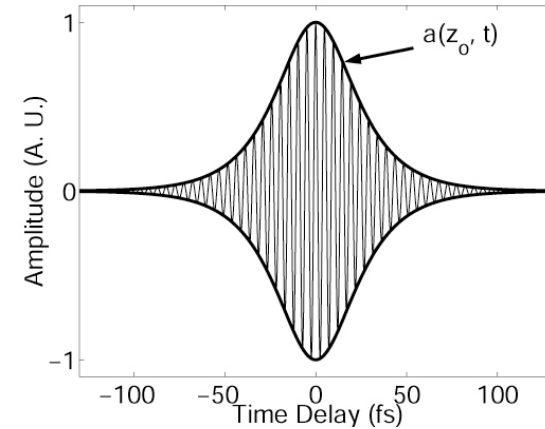
~~$\chi^{(2)}$
 2nd order
 ↓
 SHG, parametric mixing,
 electro-optics effect~~

$\chi^{(3)}$
 3rd order
 ↓
 Brillouin and Raman,
 Optical Kerr effect

$$\frac{\partial^2 \vec{E}(z, t)}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}(z, t)}{\partial t^2} = \mu_0 \frac{\partial^2}{\partial t^2} (\vec{P}_L(z, t) + \vec{P}_{NL}(z, t))$$

$$\vec{P}_{NL}(z, t) = \varepsilon_0 \chi^{(3)} : E(z, t) E(z, t) E(z, t)$$

$$E(z, t) = c \cdot a(z, t) \cdot \exp(i\beta_0 z - i\omega_0 t)$$



• Nonlinear Schrödinger equation

Describes the evolution of the pulse envelop in function of time and distance.

$$\frac{\partial a(z, t)}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 a(z, t)}{\partial t^2} + i\gamma |a(z, t)|^2 a(z, t)$$

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}} > 0 \quad \text{Kerr coefficient with } A_{eff} \text{ the area of the beam cross section}$$

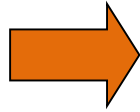
Time coordinate is shifted to eliminate the propagation delay

$$t = t_{old} - z/v_g$$

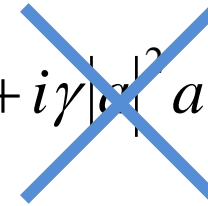
C. R. Menyuk, IEEE J. Quantum Electron. 25, 12, 2674-2682, December 1, 1989.

L. F. Mollenauer *et al.*, in *Opt. Fiber Telecommunications I VA*, (Academic, San Diego, Calif., 1997).

Low intensity : neglect nonlinear term ($|a|^2$ is low)



$$\frac{\partial a}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 a}{\partial t^2} + i \gamma |a|^2 a$$



Trivial solution in frequency domain :

$$\frac{\partial \tilde{a}}{\partial z} = i \frac{\beta_2}{2} \Omega^2 \tilde{a}$$

$$\Omega = \omega - \omega_0$$

$$\tilde{a}(\Omega, z) = \exp(i\beta_2 \Omega^2 z / 2) \tilde{a}(\Omega, 0)$$



Multiplication by a quadratic phase term : power spectrum remains unchanged

Temporal domain

Frequency domain

$$a(t,0) = A_0 \exp\left[-\frac{t^2}{2\tau_0^2}\right]$$

FT



$$\tilde{a}(\Omega,0) = A_0 \sqrt{2\pi\tau_0^2} e^{-\frac{\Omega^2}{2}\tau_0^2}$$



$\times \exp(i\beta_2\Omega^2 z / 2)$

$$a(t,z) = A(z) e^{-\frac{1+iC(z)}{2\tau(z)^2} t^2}$$

IFT



$$\tilde{a}(\Omega,z) = A_0 \sqrt{2\pi\tau_0^2} e^{-\frac{\Omega^2}{2}(\tau_0^2 - i\beta_2 z)}$$

where :

$$L_D = \tau_0^2 / |\beta_2| \quad \text{Dispersion length}$$

$$\tau(z) = \tau_0 \sqrt{1 + z^2 / L_D^2} \quad \text{Pulse width}$$

$$C(z) = \text{sign}(\beta_2) z / L_D = \beta_2 z / \tau_0^2 \quad \text{Chirp parameter}$$

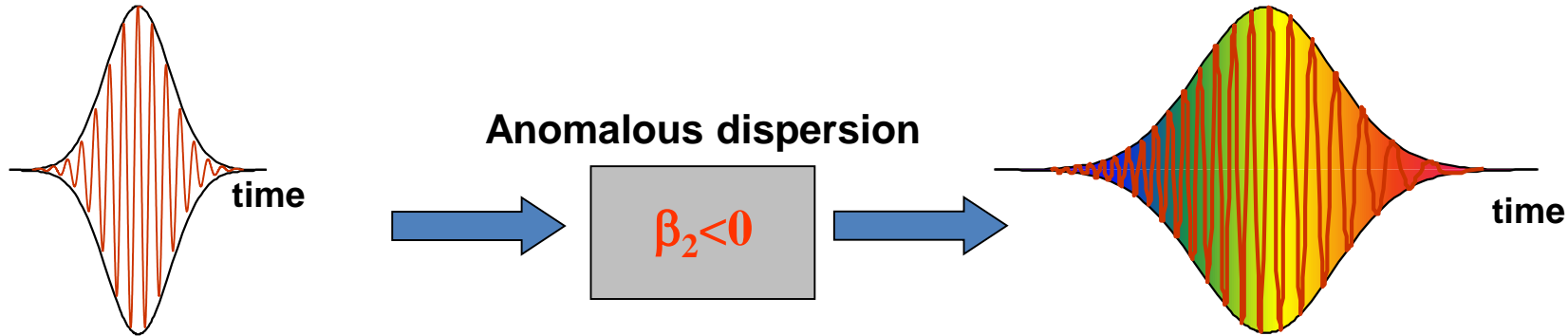
$$a(t, z) = A(z) \exp\left[-\frac{t^2}{2\tau(z)^2} + i\varphi_L(z, t)\right]$$

Parabolic phase : $\varphi_L = -\frac{C(z)}{2\tau(z)^2} t^2$

⇒ **Linear evolution of instantaneous frequency :** $\delta\omega_L(t, z) = -\frac{\partial\varphi_L}{\partial t} = \text{sign}(\beta_2) \frac{z/L_D}{\tau(z)^2} t$

• **Temporal domain : pulse stretching**

The blue components propagate faster and arrive first



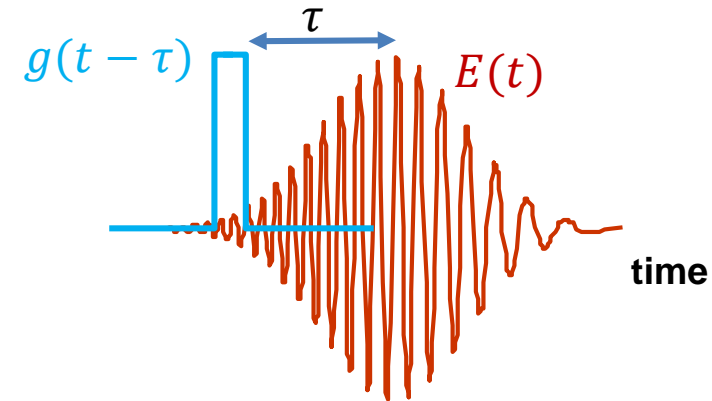
• **Spectral domain : the spectrum remains unchanged**



Frequency Resolved Optical Gating (FROG)

→ Measure the spectrogram given by :

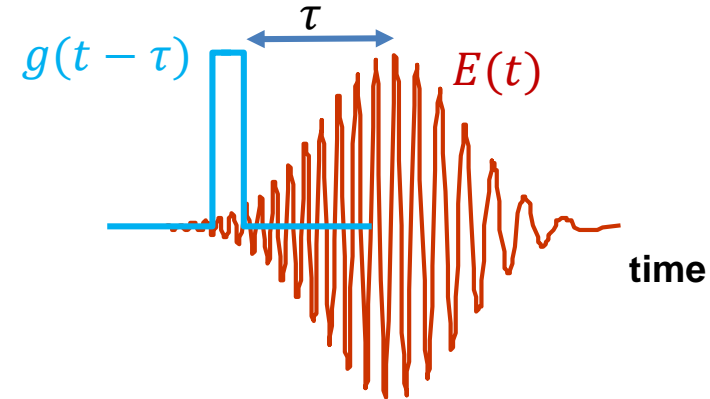
$$S(\omega, \tau) = \left| \int_{-\infty}^{+\infty} E(t)g(t - \tau) e^{i\omega t} dt \right|^2$$



Frequency Resolved Optical Gating (FROG)

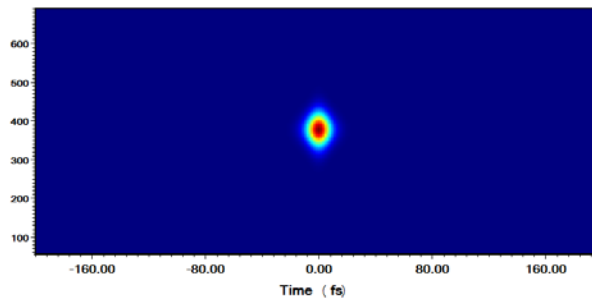
→ Measure the spectrogram given by :

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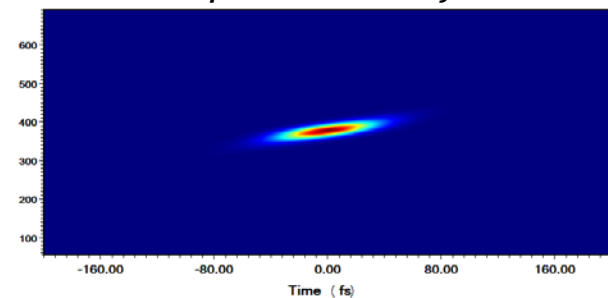


Spectrograms of a 10 fs pulse @ 800 nm

$$\varphi^{(2)} = 0$$



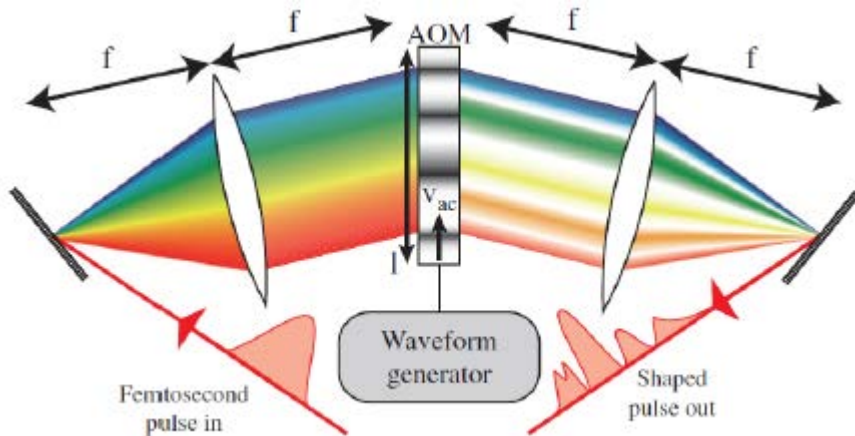
$$\varphi^{(2)} = 200 \text{ fs}^2$$



→ Iterative algorithms to retrieve the spectral phase distribution $\varphi(\omega)$

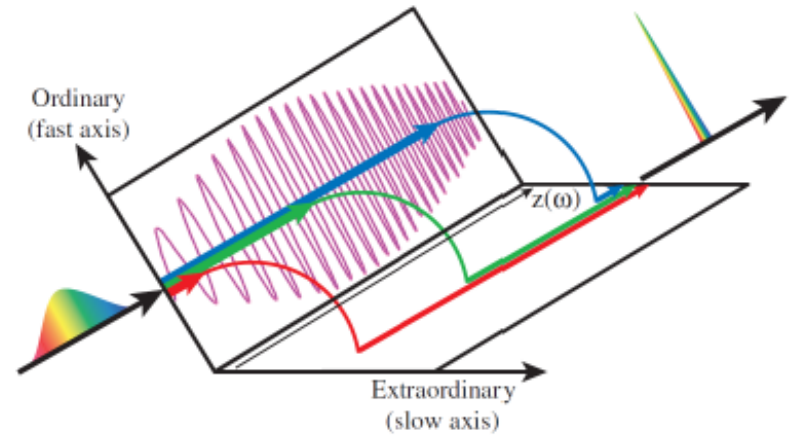
Modify the frequency phase to control the temporal shape

4f phase shaper



Discrete shaping of the phase by the AOM placed in the Fourier plane.

Dazzler



Shaping of spectral phase through chirp of acoustic wave.

Montmayrant & Blanchet, *J. Phys. B: At. Mol. Opt. Phys.* **43** 103001 (2010)

Short propagation length : we neglect the dispersion term

$$\frac{\partial a}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 a}{\partial t^2} + i\gamma |a|^2 a$$



$$\frac{\partial a}{\partial z} \approx +i\gamma |a|^2 a(t, z)$$

Approached solution can be found assuming a constant power over dz :

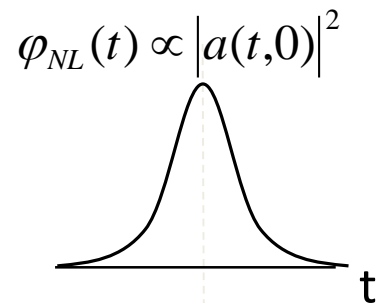
$$\frac{\partial |a(t, z)|^2}{\partial z} = 0 \quad \Rightarrow \quad |a(t, z)|^2 = |a(t, 0)|^2$$

$$\Rightarrow \quad a(t, z) = e^{+i\gamma |a(t, 0)|^2 z} a(t, 0)$$

$$a(t, z) = e^{+i\gamma|a(t,0)|^2 z} a(t, 0) = e^{+i\varphi_{NL}(t,z)} a(t, 0)$$

The pulse acquires chirp :

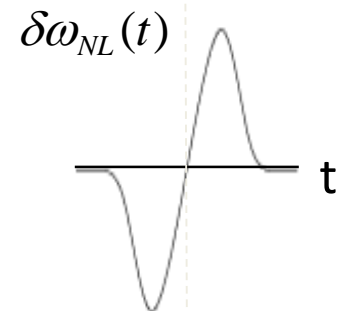
$$\delta\omega_{NL} = -\frac{\partial\varphi_{NL}}{\partial t} = -\gamma \frac{\partial|a(t,0)|^2}{\partial t} z = -\frac{2\pi n_2 z}{\lambda A_{eff}} \frac{\partial|a(t,0)|^2}{\partial t}$$



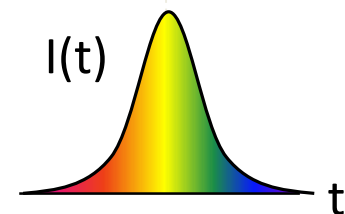
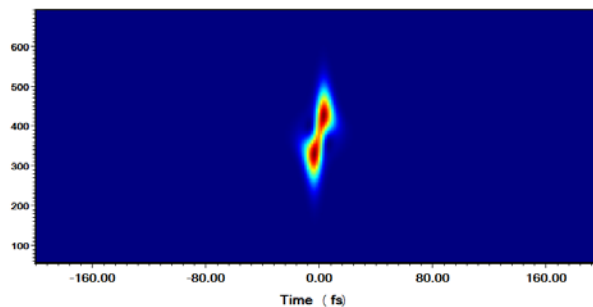
➡ *New frequencies are created*

➡ *For $n_2 > 0$: frequencies distribution similar to normal dispersion.*

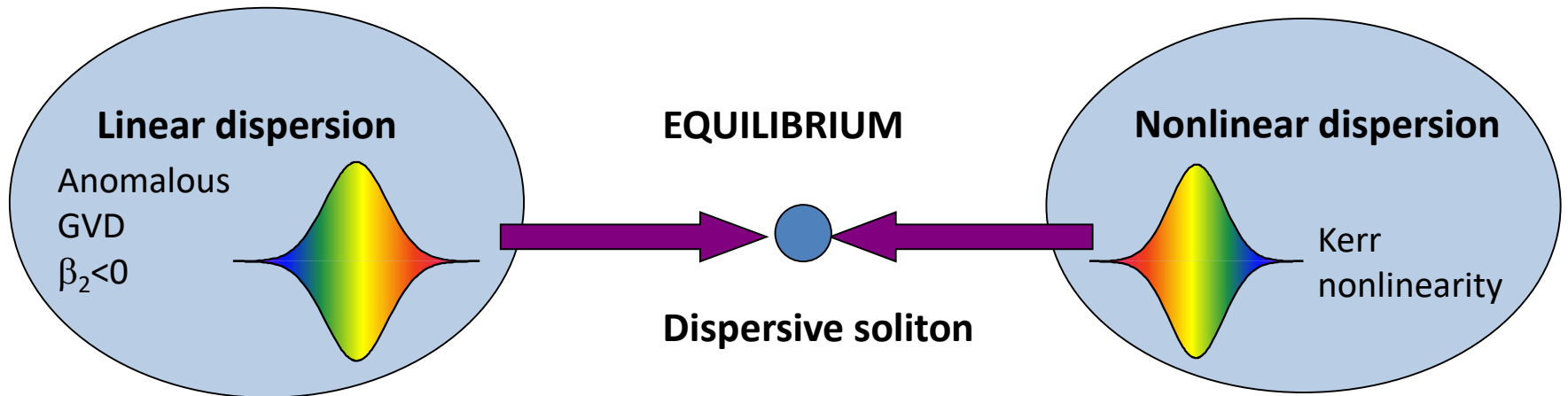
➡ *Nonlinear chirp could be compensated by anomalous dispersion.*



Propagation along 2 m fiber



$$\frac{\partial a(z,t)}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 a(z,t)}{\partial t^2} + i\gamma |a(z,t)|^2 a(z,t)$$



Dispersion & nonlinearity compensate exactly for an hyperbolic secant pulse profile :

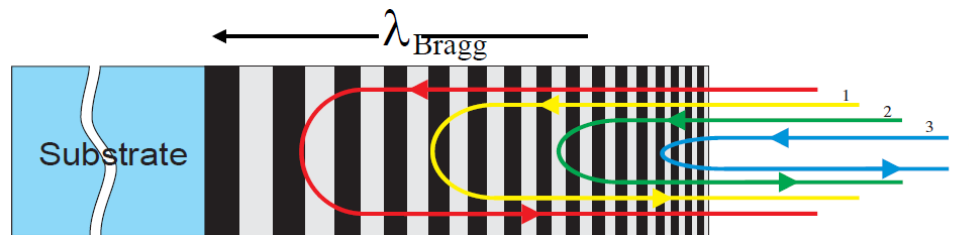
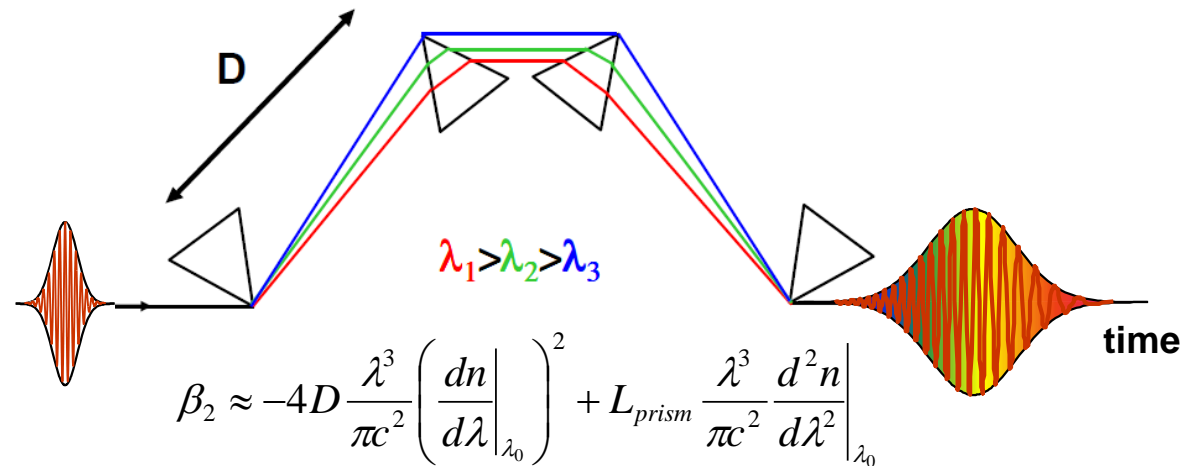
$$a(t) = \text{sech}(t / \tau_p) \exp(iz / z_{sol})$$

Ideal medium : homogeneous, isotropic and transparent!

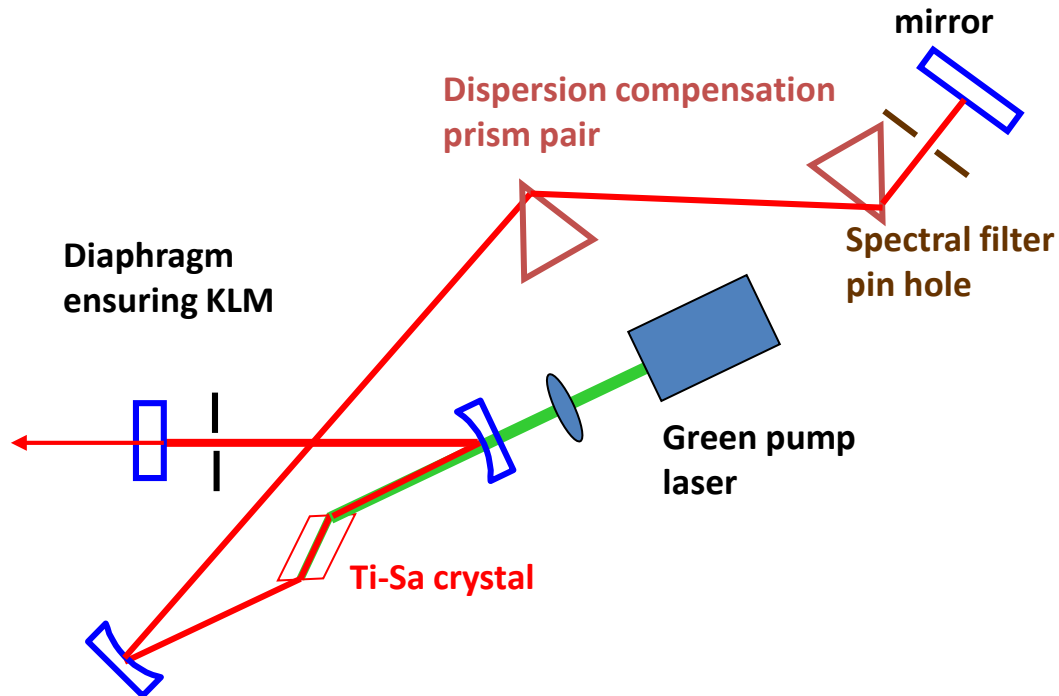
Zakharov and Shabat, Sov. Phys. JETP 34, 62 (1972), Hasegawa and Tappert (1973), APL 23, 142 (1973)

Glass materials : positive (normal) GVD in the visible and near infrared.

Dispersion management systems : prism pairs, grating pairs, chirped mirrors, GTI mirrors, chirped Bragg gratings.



R. Szipöcs et al., *Opt. Lett.* 19, 201 (1994)



Typical performances:

- Pulse duration < 100 fs
- Tunable from 700 nm to 1080 nm
- Energy = 10 nJ
- Repetition rate : MHz

<https://www.spectra-physics.com>



<https://fr.coherent.com>



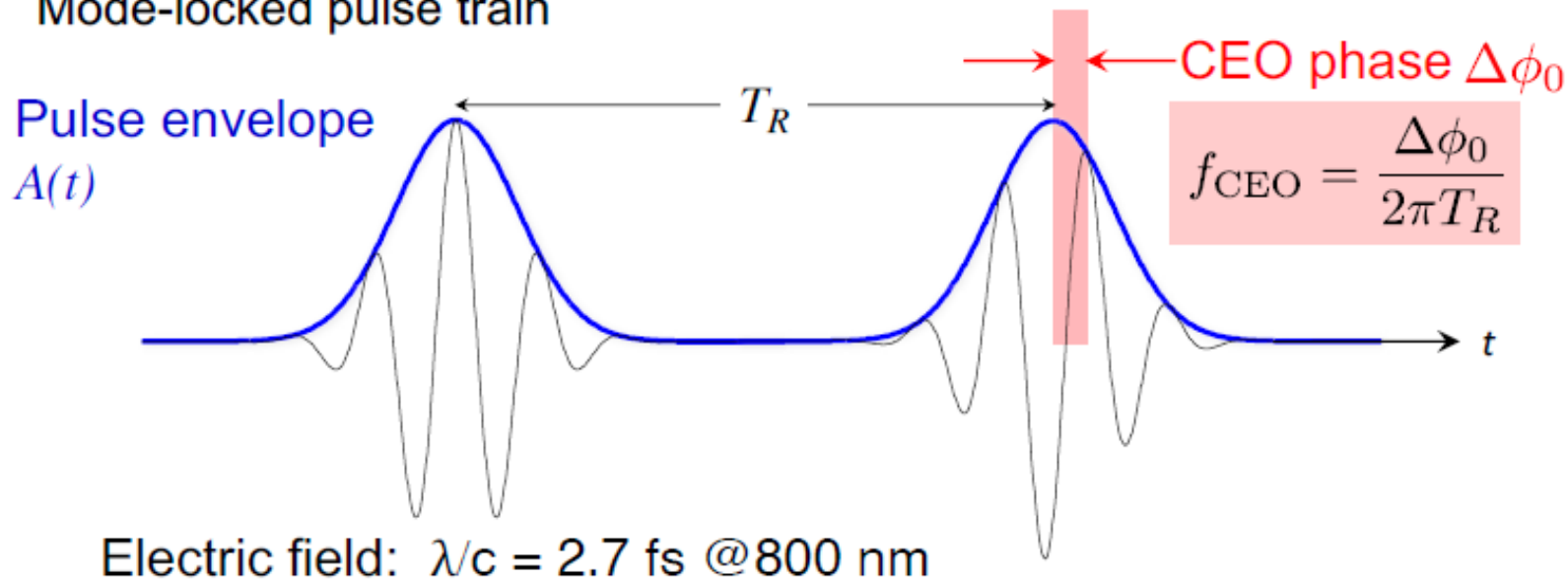
<https://www.laserquantum.com>



<https://www.thorlabs.com>



Mode-locked pulse train



Need to stabilize repetition rate and carrier envelop offset phase

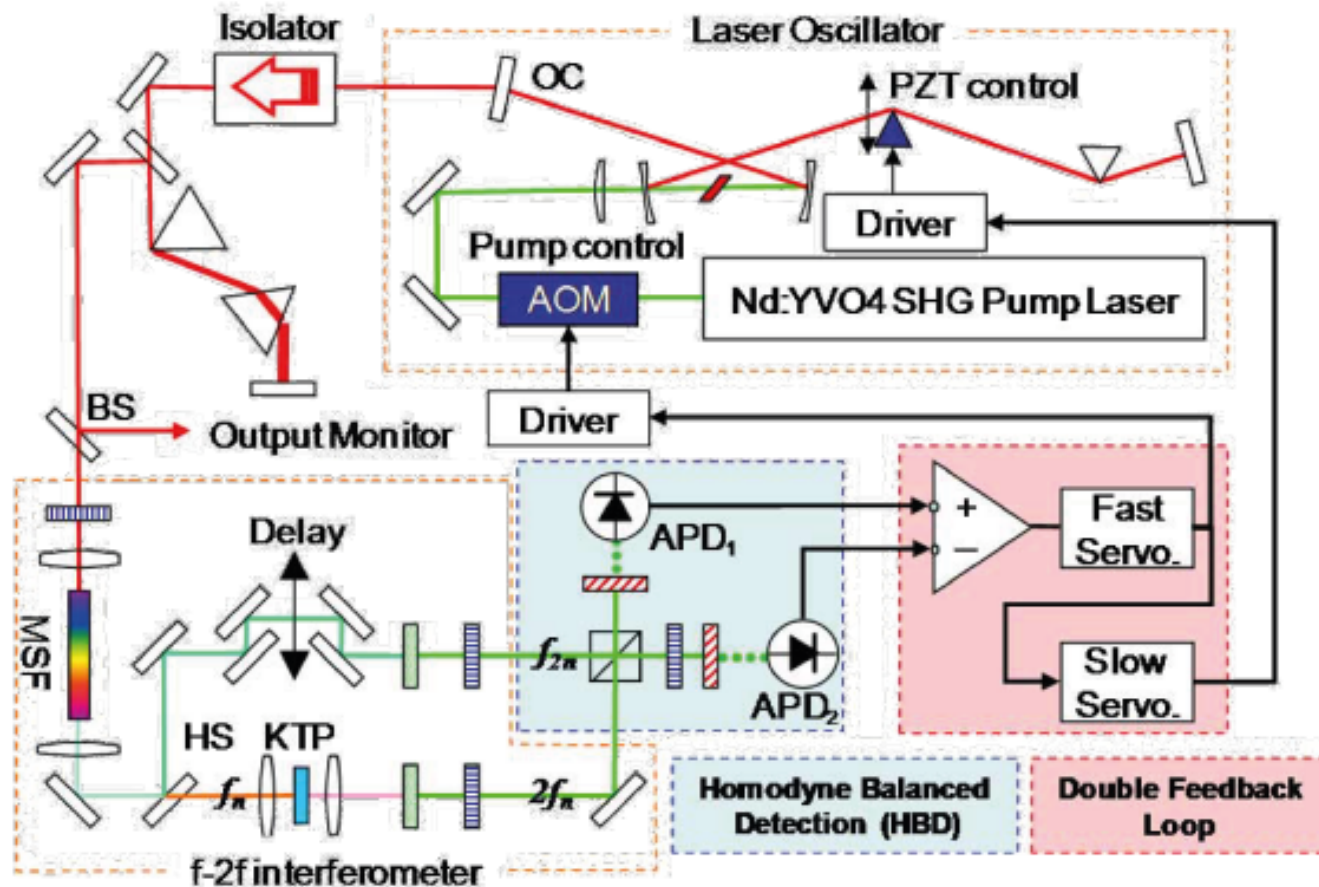
$$f_R = \frac{1}{T_R} = \frac{v_g}{2l_{las}}$$

Cavity length control

$$f_{CEO} = \left(1 - \frac{v_g}{v_\phi}\right) \cdot f_{las}$$

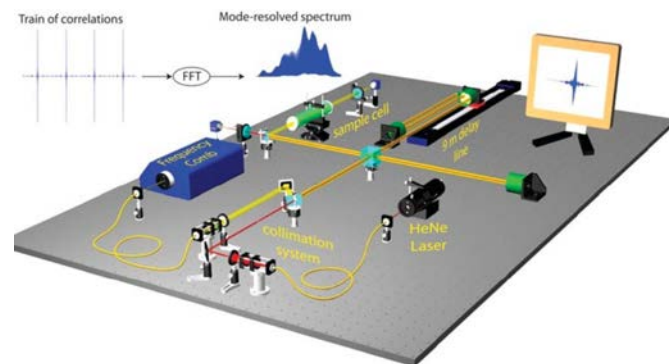
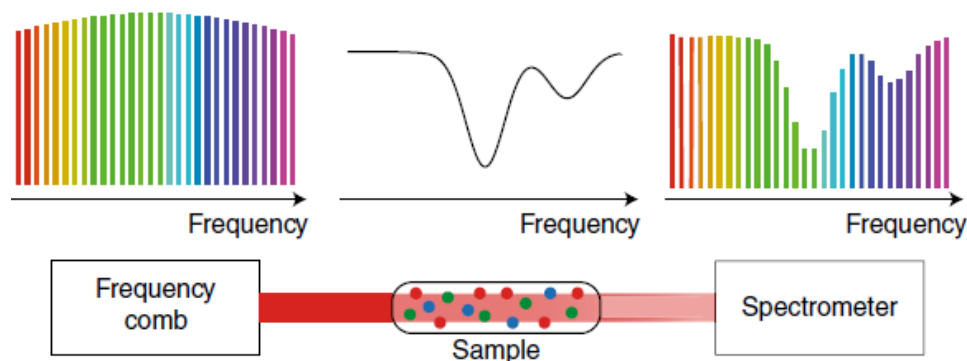
Phase and group velocities control

f-2f interferometer for CEO phase measurements



D. J. Jones et al. Science **288**, 635 (2000), A. Apolonski et al., Phys. Rev. Lett. **85**, 740 (2000)

- Spectrum of a femtosecond laser pulse consists of millions of sharp lines
- These lines are equidistant across the entire spectrum
- A femtosecond laser is a “ruler” for frequencies !

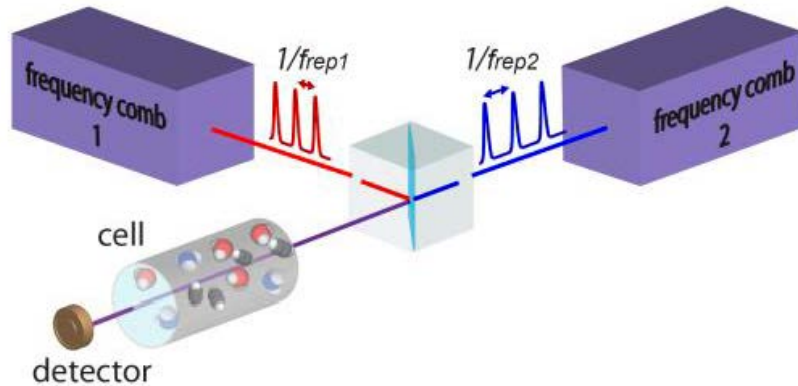


Prix Nobel 2005 – J. L. Hall et T. W. Hänsch

Need for FTIR or VIPA to resolve the comb components!

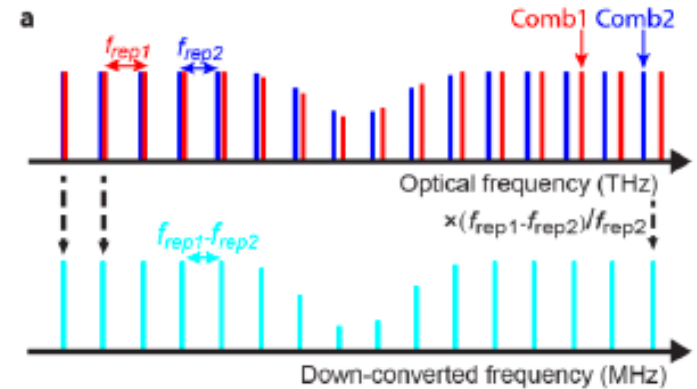
T. Udem, R. Holzwarth, T. W. Hänsch, *Nature* **416**, 233 (2002), *Nature Photonics* **volume 13**, pages146–157 (2019) , S. A. Diddams et al., *Nature*, vol. 445 (2007),

Dual comb spectroscopy

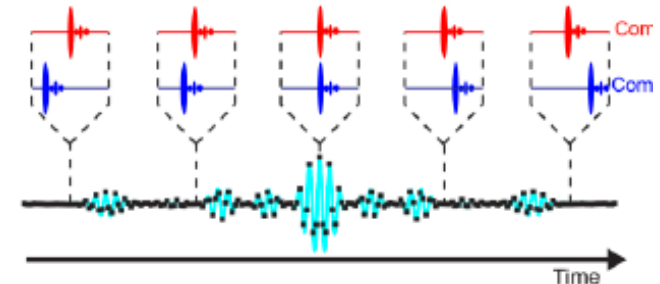


- Combine two optical frequency combs
- Intensity beat on photodetector
- Down-conversion to radio frequencies (RF)

Frequency domain



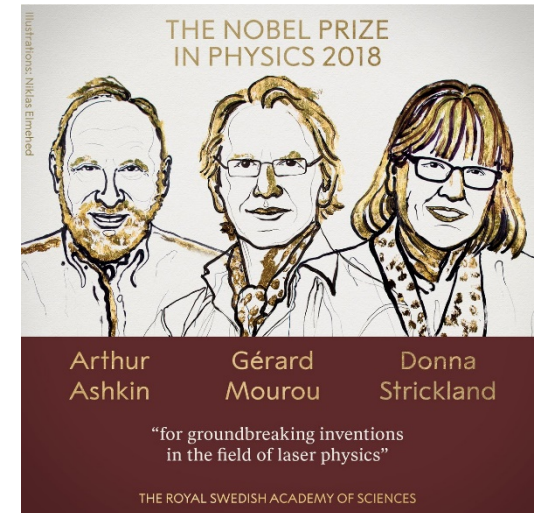
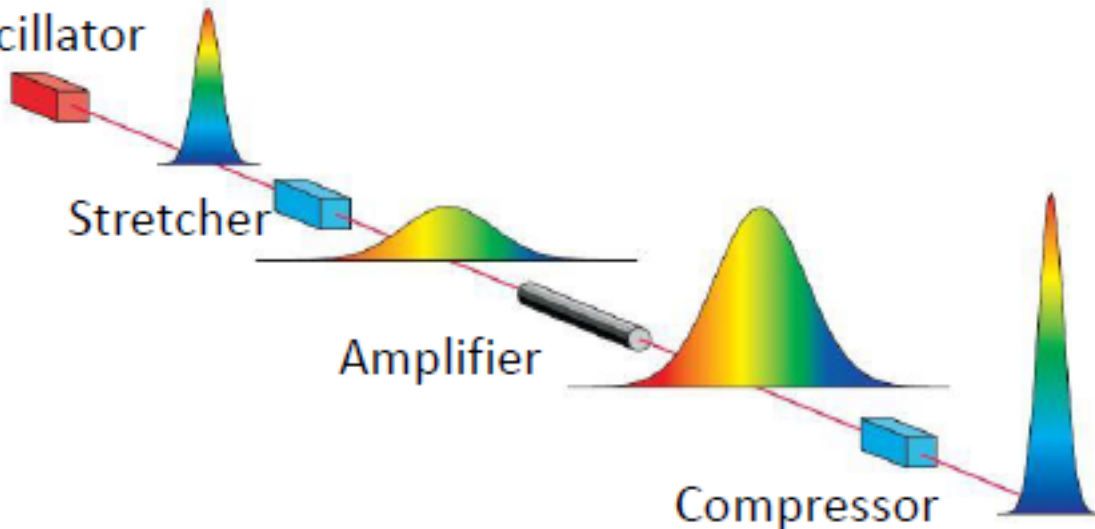
Time domain



T. H. Hänsch, N. Picqué, Jour. of Phys.: Conf. Series 467 (2013), G. Millot et al., Nat. Photon. **10**, 27–30 (2016)

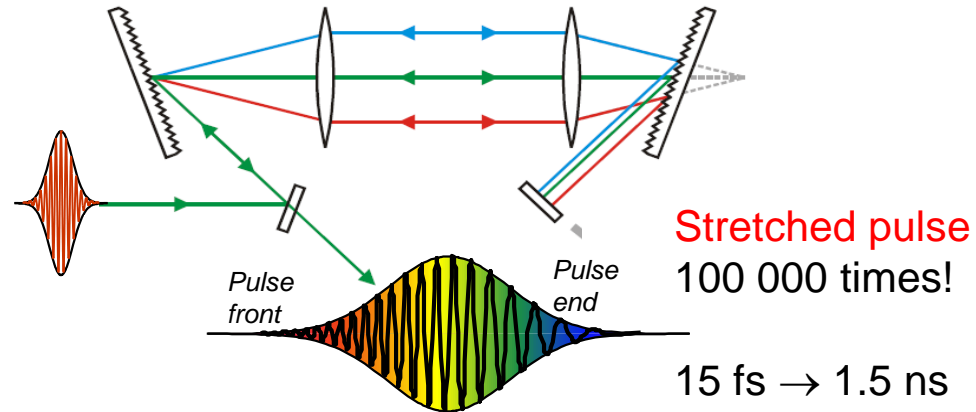
Energy scaling concept Minimize impact of nonlinear effects

Laser-
oscillator

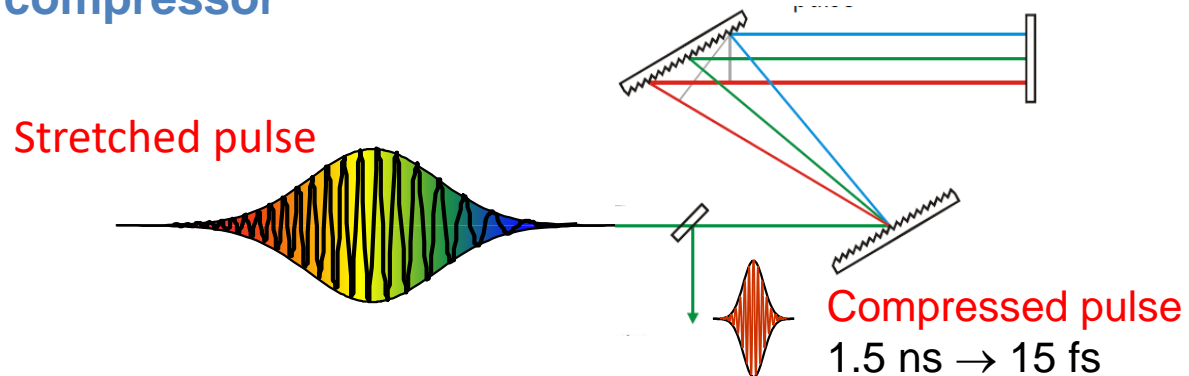


D. Strickland and G. Mourou, *Opt. Commun.* **56**, 219–221 (1985).

Need for large stretching ratios : grating-based stretcher

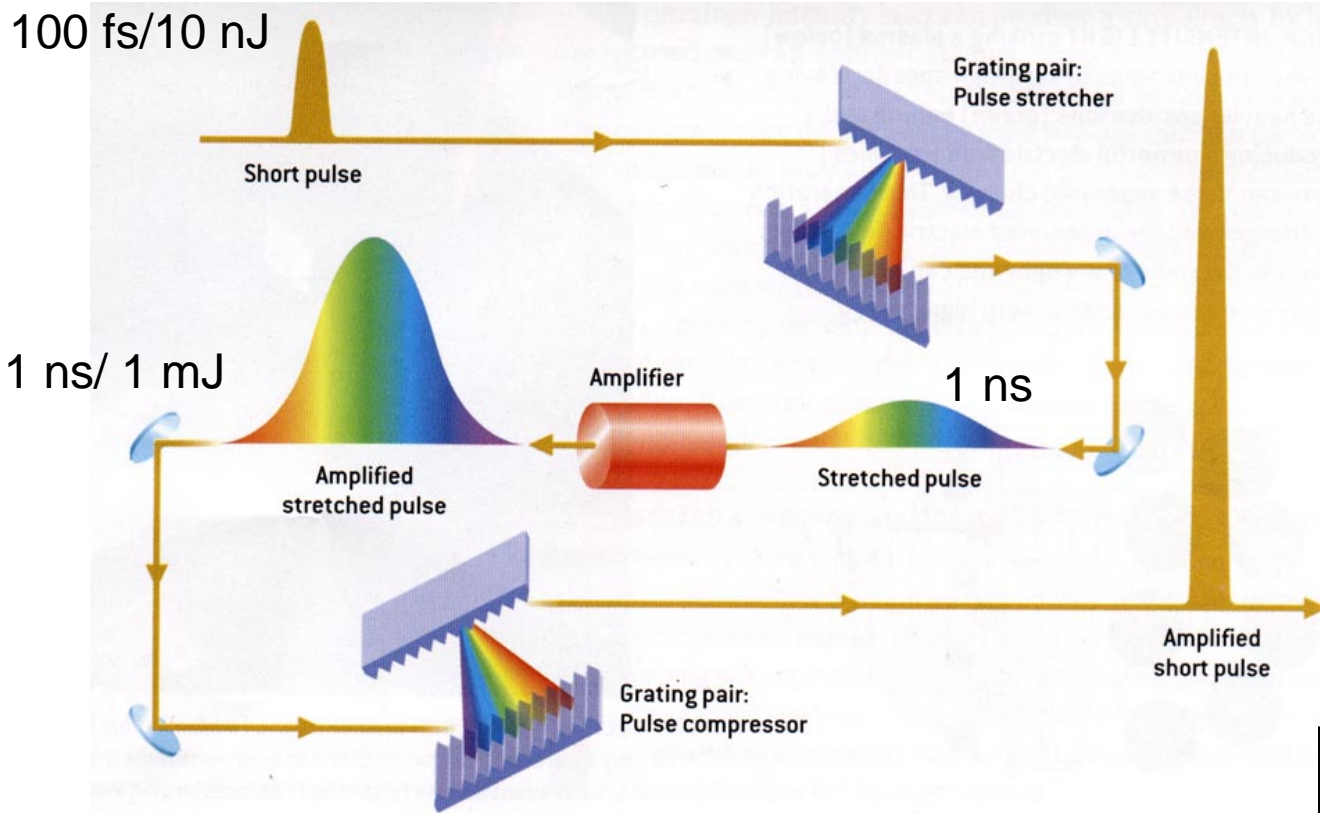


Grating-based compressor



E. B. Treacy, *IEEE JQE* 5, 454 (1969); Fork et al., *Opt. Lett.* 9, 150 (1984)

Amplification à dérive de fréquence (CPA)



$$\Delta T = 100 \text{ fs}$$

$$E = 1 \text{ mJ}$$

Peak power :

$$P_{peak} = \frac{E}{\Delta T}$$

Focalisation to small spot « A »

$$I_{peak} = \frac{P_{peak}}{A}$$

$$A \sim 10 \mu\text{m}^2$$

$$P_{cr\hat{e}te} = 10 \text{ GW}$$

$$I_{cr\hat{e}te} = 10^{17} \text{ W/cm}^2$$

Standard commercial products



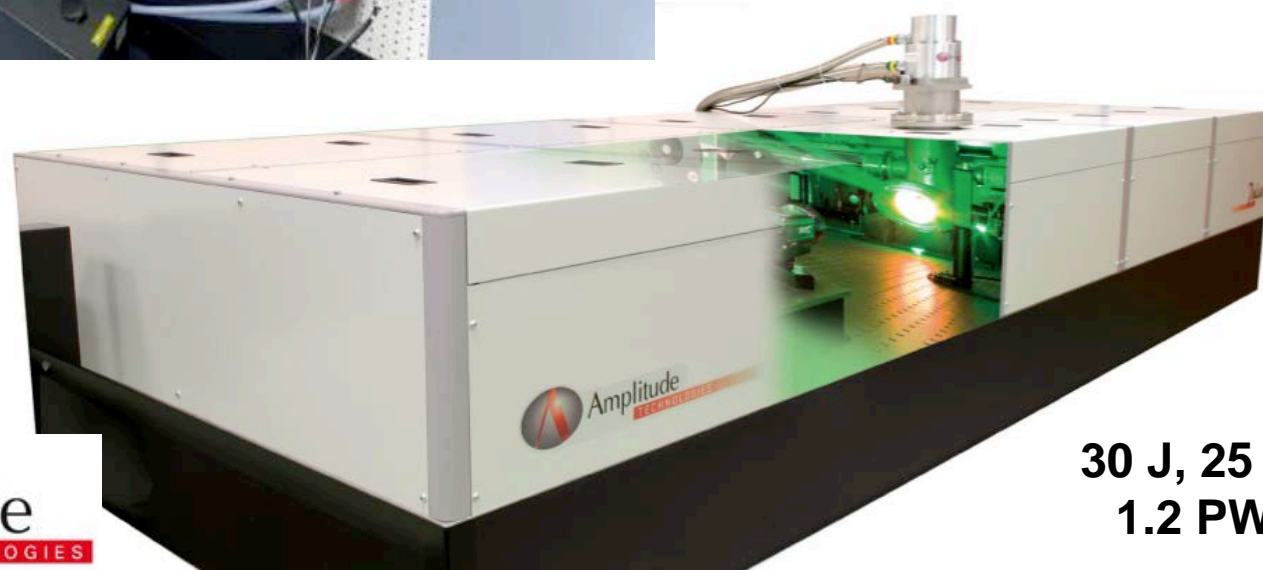
Ti-Sa lasers : 35 – 120 fs, multi-mJ @ 1 Khz





THALES
BELLA

40 J, 30 fs
1.3 PW



30 J, 25 fs
1.2 PW

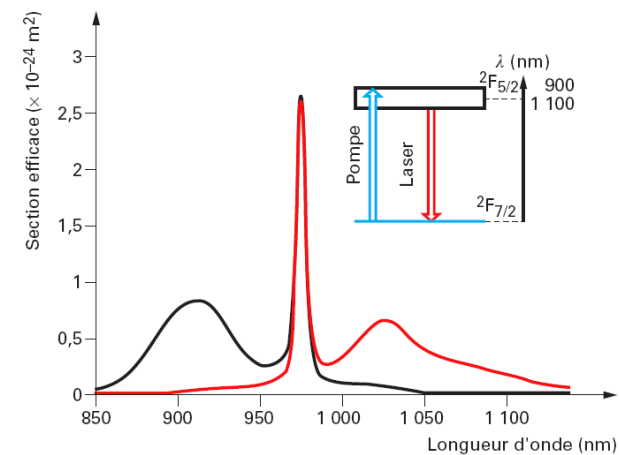
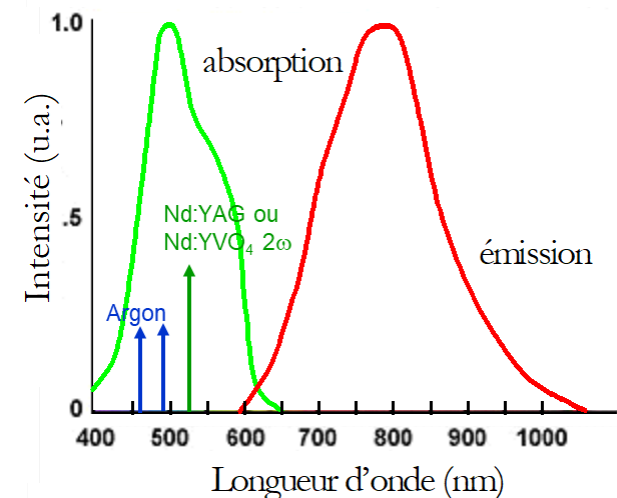


Limitation of titanium-sapphire laser systems

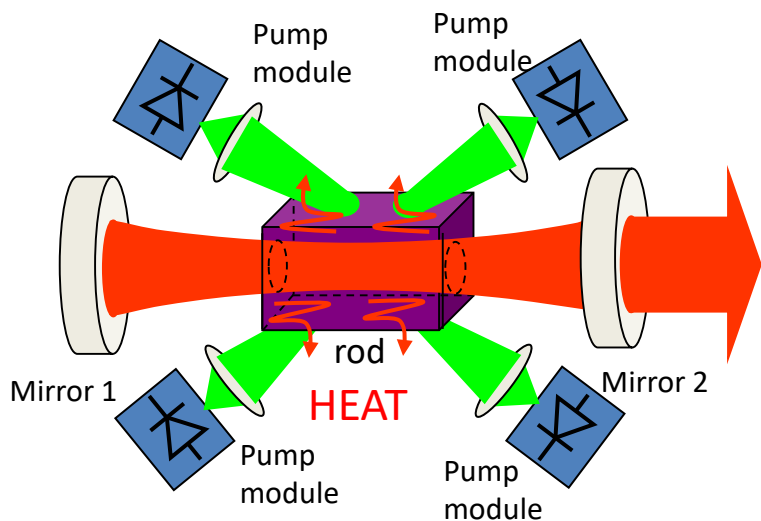
- ☹️ **Low efficiency : pumping in the green**
- ☹️ **Thermal management**
- ☹️ **Complexity and cost**

Ytterbium-doped host materials

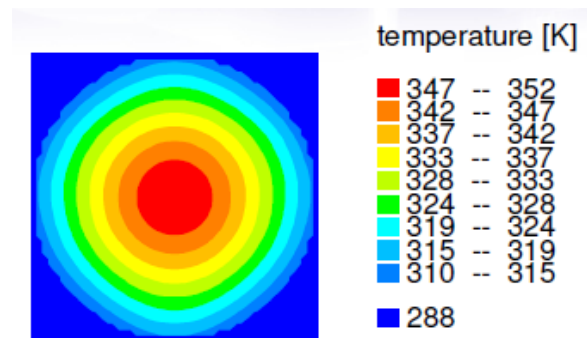
- 😊 **Low quantum defect**
- **Good thermal conductivity**
- **Large gain bandwidth**
- 😊 **Diode pumping at 980 nm**



Conventional laser

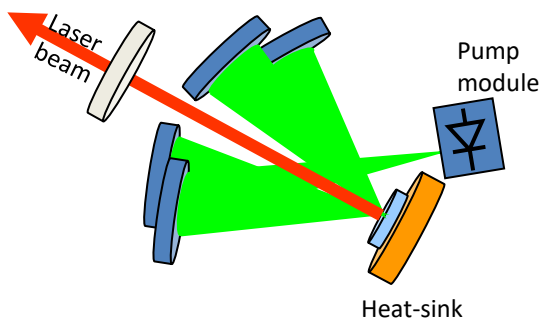


⇒ power dependent thermal lensing and thermal stress-induced birefringence

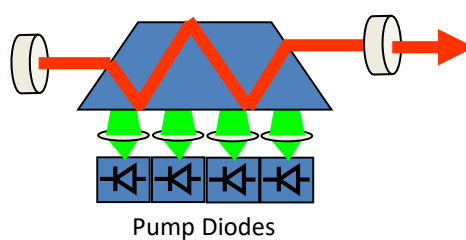


Solutions to reduce thermo-optical issues

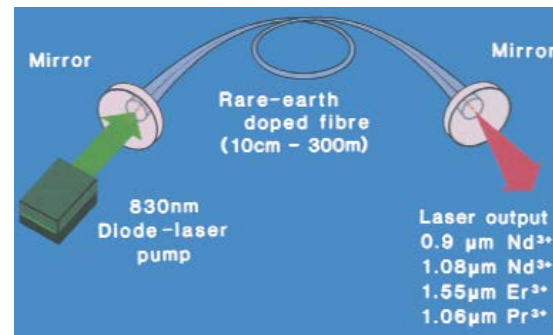
Thin-disk lasers



Slab lasers



Fibre lasers





Yb-lasers : 300- 500 fs, $>100\text{-}\mu\text{J}$ @ >100 kHz

